mathematical biology, we have provided a unified picture of what happened on the Millennium Bridge five years ago, both for the bridge vibrations and the crowd dynamics. The approach suggested here may also prove useful for estimating the damping needed to safeguard other bridges, present and future, against synchronous lateral excitation by pedestrians.

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1. Dallard, P. *et al. Struct. Eng.* **79**, 17–33 (2001).
2. Dallard, P. *et al. J. Bridge Eng.* **6**, 412–417 (2001).
3. McRobie, A., Morgenthal, G., Lasenby, J. & Ringer, M. *Proc. Inst. Civ. Eng. Bridge Eng.* **156**, 71–79 (2003).
4. Roberts, T. M. *Proc. Inst. Civ. Eng. Bridge Eng.* **156**, 155–160 (2003).
5. Newland, D. E. *Proc. Inst. Mech. Eng.* **218**, 477–492 (2004).
6. Nakamura, S. *J. Struct. Eng.* **130**, 32–37 (2004).
7. Glass, L. & Mackey, M. C. *From Clocks to Chaos: The Rhythms of Life* (Princeton Univ. Press, Princeton, 1988).
8. Winfree, A. T. *The Geometry of Biological Time* 2nd edn (Springer, New York, 2001).
9. Kuramoto, Y. *Chemical Oscillations, Waves and Turbulence* (Springer, Berlin, 1984).
10. Strogatz, S. H. *Physica D* **143**, 1–20 (2000).

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NANOSCALE HYDRODYNAMICS

Enhanced flow in carbon nanotubes

Nanoscale structures that could mimic the selective transport and extraordinarily fast flow possible in biological cellular channels would have a wide range of potential applications. Here we show that liquid flow through a membrane composed of an array of aligned carbon nanotubes is four to five orders of magnitude faster than would be predicted from conventional fluid-flow theory. This high fluid velocity results from an almost frictionless interface at the carbon-nanotube wall.

Biological channels act as chemically selective gatekeepers and have protein walls that allow extremely rapid transit[1]. Nanometre-scale pores with chemical selectivity have been prepared[2,3] but fluid flow through them is slow: this limitation is predicted by the Hagen–Poiseuille equation and is because conventional laminar flow has zero fluid velocity at the pore walls.

In theory, the flow of molecules inside carbon nanotubes could be much faster. Water should be able to flow fast through hydrophobic single-walled carbon nanotubes because the process creates ordered hydrogen bonds between the water molecules[4]. Ordered hydrogen bonds between water molecules and the weak attraction between the water and smooth carbon-nanotube graphite sheets should then result in almost frictionless and very rapid flow[4]. If a theoretical volume rate comparable to that of the protein channel aquaporin-1 (ref. 4) is divided by the carbon-nanotube cross-sectional area, the expected water flow velocity is about 90 cm s^{-1}. Fast flow velocities are also predicted just from the frictionless nature of the carbon-nanotube walls[5] and from the rapid diffusion of hydrocarbons[6,7].

To realize these high flow velocities, we used a freshly fabricated membrane consisting of aligned multiwalled carbon nanotubes, with graphitic inner cores (diameter about 7 nm) and a high area density (5×10^{10} per cm^2), crossing a solid polystyrene film[8]. We measured the flow of water and a variety of solvents through this membrane at about 1 atm applied pressure (Table 1). In a control experiment, we verified that no macroscopic defects were present in the membrane and determined the available pore area (see supplementary information).

We found that the flow rates are four to five orders of magnitude faster than conventional fluid flow would predict through pores of 7 nm diameter. Contrary to predictions based on hydrodynamics, the flow rate does not decrease with increased viscosity (compare hexane and water in Table 1). The results also indicate that flow velocity, when adjusted for differences in viscosity, increases for more hydrophilic fluids. The flow of hydrogen-bonded fluids decreases after a few minutes, but this does not occur with alkanes or aqueous solutions of potassium chloride. Reduction in the flow of associated liquids (water and alcohols) with time can be attributed to flow-induced solvent ordering or the formation of bubbles (our unpublished results).

Table 1 | Pressure-driven flow through aligned MWCNT membrane

Liquid	Initial permeability*	Observed flow velocity†	Expected flow velocity†	Slip length (nm)
Water	0.58	25	0.00057	54
	1.01	43.9	0.00057	68
	0.72	9.5	0.00015	39
Ethanol	0.35	4.5	0.00014	28
iso-Propanol	0.088	1.12	0.00077	13
Hexane	0.44	5.6	0.00052	9.5
Decane	0.053	0.67	0.00017	3.4

MWCNT, multiwalled carbon nanotube. For details of methods, see supplementary information. *Units, cm^3 per cm^2 min bar. †Flow velocities in cm s^{-1} at 1 bar. Expected flow velocity is that predicted from conventional flow.

We conclude that these high fluid velocities are possible because of a frictionless surface at the carbon-nanotube wall. This result could be explained in conventional terms of slip lengths, which are remarkably long. The slip length is an extrapolation of the extra pore radius required to give zero velocity at a hypothetical pore wall (the boundary condition for conventional materials). The observed slip lengths (3–70 μm) are much longer than the pore radius (3.5 nm) that is consistent with a nearly frictionless interface. The slip length decreases as solvents become more hydrophobic (Table 1), which indicates stronger interaction with the carbon-nanotube wall. The observed flow velocities for water (10–44 cm s^{-1}) are close to the extrapolated rate predicted for water flow through single-walled carbon nanotubes (about 90 cm s^{-1})[4]. Butane flows through carbon nanotubes at about 26 cm s^{-1} (ref. 6), which is consistent with our measurement for hexane.

These results show that the speed of fluid flow through the aligned carbon-nanotube membrane approaches that through biological channels. The membrane fabrication is scalable to large areas, which could be useful industrially for chemical separations; chemical functionality is near the core entrance[9] and each side of the membrane can be independently modified with different functional groups[10]. These advantages also make the aligned carbon-nanotube membrane a promising mimic of protein channels for transdermal drug delivery and selective chemical sensing.

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1. Hille, B. *Ionic Channels of Excitable Membranes* (Sinauer, Sunderland, Massachusetts, 1984).
2. Jirage, K. B., Hulteen, J. C. & Martin, C. R. *Science* **278**, 655–658 (1997).
3. Klein, E. *J. Membr. Sci.* **179**, 1–27 (2000).
4. Hummer, G., Rasaih, J. C. & Noworyta, J. P. *Nature* **414**, 188–190 (2001).
5. Sokhan, V. P., Nicholson, D. & Quirke, N. *J. Chem. Phys.* **117**, 8531–8539 (2002).
6. Skoulidas, A. I., Ackerman, D. M., Johnson, J. K. & Sholl, D. S. *Phys. Rev. Lett.* **89**, 185901 (2002).
7. Mao, Z. & Sinnott, S. B. *J. Phys. Chem. B* **105**, 6916–6924 (2001).
8. Hinds, B. J. *et al. Science* **303**, 62–65 (2004).
9. Majumder, M., Chopra, N. & Hinds, B. J. *J. Am. Chem. Soc.* **127**, 9062–9070 (2005).
10. Chopra, N., Majumder, M. & Hinds, B. J. *Adv. Funct. Mater.* **15**, 858–864 (2005).

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